Code No. 7960/CDE/N

FACULTY OF SCIENCE

M.Sc. (Final) CDE Examinations, July 2019

Subject: Mathematics

Paper- I : Complex Analysis

Time: 3 Hours **Max. Marks: 80**

PART – A (5 x 4 = 20 Marks)

(Short Answer Type)

Note: Answer any five of the following questions. Each question carries 4 marks.

1. Find the fixed points of the linear transformation $w = \frac{3z-4}{z-1}$
2. Find the linear transformation which carries 0, i, -i into 1, -1, 0 respectively.
3. Compute $\int_{\text{¥}} x dz,$ where γ is the directed line segment from 0 to 1+i.
4. If $f(z)$ and $g(z)$ are analytic in a region Ω, and if $f(z) = g(z)$ on a set which has an accumulation point in Ω, then prove that $f(z) = g(z)\ \forall\ z \in \Omega$
5. Find the poles and residues of the function $f(z) = \frac{1}{z^2 + 5z + 6}$
6. Find the number of roots of the equation $z^7 - 2z^5 + 6z^3 - z + 1 = 0$ in $|z| < 1$
7. Express $\sum_{n=-\infty}^{\infty} \frac{1}{z^2 - n^2}$ in the closed form
8. Show that $\Gamma\left(\frac{1}{6}\right) = 2^{-\frac{1}{3}}\left(\frac{3}{\text{®}}\right)^{\frac{1}{2}}\Gamma\left(\frac{1}{3}\right)^2$

PART – B (12 x 5 = 60 Marks)

(Essay Answer Type)

Note: Answer all the following questions by using internal choice. Each question carries 12 marks.

9. a) State and prove Lucas theorem.

OR

b) Find the radius of convergence of the following power series:

$\sum n!z^n, \sum q^{n^2} z^n (|q| < 1), \sum z^{n!}$

10. a) If the function $f(z)$ is on the rectangle $R = \{(x, y): a \le x \le b, c \le y \le d\}$, then prove that $\int_{\partial R} f(z) dz = 0$

OR

b) State and prove the Taylor's theorem.

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11. a) If $f(z)$ is meromorphic in a region Ω with the zeros a_j, and the poles b_k, then prove that

$$\frac{1}{2\text{®}i}\int_{\text{¥}}\frac{f'(z)}{f(z)}dz=\sum_j n(\text{¥},a_j)-\sum_k n(\text{¥},b_k)$$

For every cycle γ which is homologous to zero in Ω and does not pass through any of the zeros on poles.

OR

b) Evaluate $\int_o^{\frac{\text{®}}{2}}\frac{dx}{a+\sin^2 x}\ \ (|a|>1)$

12. a) State and prove Laurent's theorem.

OR

b) Prove that $(2\text{®})^{\frac{n-1}{2}}\Gamma(z)=n^{z-\frac{1}{2}}\Gamma\left(\frac{z}{n}\right)\Gamma\left(\frac{z+1}{n}\right)\text{............}\Gamma\left(\frac{z+n-1}{n}\right).$

13. a) State and prove Jensen's formula.

OR

b) State and prove Schwarz's theorem.

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Code No. 7975/CDE

FACULTY OF SCIENCE

M.Sc. (Final) CDE Examination, July 2019

Subject : Statistics

Paper – I : Statistical Inference

Time : 3 Hours **Max. Marks: 80**

Note: Answer all questions from Part – A and Part – B. Each question Carries 4 marks in Part – A and 12 marks in Part – B.

PART – A (5x4 = 20 Marks)
(Short Answer Type)

1. Define family of distribution with monotone likelihood ratio (MLR). Give an example of a distribution having MLR property and an example of a distribution which does not satisfy MLR property.
2. A sample of size one is taken from Poisson (λ). To test H_o: λ = 2 against H_1: λ = 1 consider the test function $\phi(x)=\begin{cases}1 & if\ x\le 2\\ 0 & otherwise\end{cases}$. Find the probability of type I, type II errors and power of the test.
3. State and prove Wald's fundamental identity.
4. Explain SPRT test procedure to test $H_o:\theta=\theta_o$ against the alternative $H_1 : \theta=\theta_1$ $(\theta>\theta_o)$for exponential distribution with mean θ.
5. Describe Wilcoxon Signed rank test.
6. Explain Chi-square test for goodness of fit.
7. Explain Hodges-Lehmann deficiency with an example.
8. Distinguish between Minimax and Bayes' decision functions. Describe admissible and optimal decision functions.

PART – B (5x12=60 Marks)
(Essay Answer Type)

9. (a) Define uniformly most powerful (UMP) test of a hypothesis. Based on a sample of size n from a distribution with pdf $f_\theta(x)=\theta x^{\theta-1}$, 0<x<1, θ>0, find a UMP size α test for testing $H_o:\theta\le\theta_o$ against $H_o:\theta>\theta_o$.

OR

(b) Describe likelihood ratio test (LRT). Establish relation between LRT and Neyman-Pearson test. Obtain asymptotic distribution of LRT.

10. (a) Obtain the bonds for constant of SPRT in terms of its strength (α, β). Develop an SPRT of strength (α, β) for testing H_o: $\sigma^2=\sigma_0^2$ against $H_1 : \sigma_2=\sigma_1^2$ where successive observations come from N(0, σ^2). Obtain OC function of the test.

OR

(b) Define OC and SN functions of SPRTY. Find the expressions for OC and ASN functions of SPRT with respect to the distribution P(X=-1)=θ, P(X=1)=1-θ, 0<θ<1.

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11. (a) Describe two-sample location problem. Discuss in detail Mann-Whitney-Wilcoxon test for two-sample location problem.

OR

(b) Describe Spearman's and Kendalls tests of independence.

12. (a) Prove that, under squared error loss function, mean of the posterior distribution is a Bayes estimator. Suppose X is distributed as $B(1,\theta)$ and prior distribution of θ is $\pi(\theta)=1$, find the Bayes estimator of θ under squared error loss function.

OR

(b) Define risk function under squared error loss function. Find Bayes risk if X has $B(n,\theta)$ distribution and prior distribution of θ is $Beta(\alpha, \beta)$.

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OU - coe OU - coe

Code No. 7965/CDE/BL

FACULTY OF SCIENCE

M.Sc. (Final) CDE Examinations, July 2019

Subject: Mathematics

Paper- I : Topology & Functional Analysis

Time: 3 Hours **Max. Marks: 100**

PART – A

(Short Answer Type)

Note: Answer any five of the following questions, choosing atleast two from each Part. All questions carry equal marks.

1. Let X be a non-empty set and let there be given a ‘closure’ operator which assigns to each subset A of X a subset $\overline{A}$ of X in such a manner that

   $(a)\ \overline{\text{¤}} = \text{¤} \quad (b) A \subseteq \overline{A} \quad (c) \overline{\overline{A}} = \overline{A} \quad (d) \overline{A \bigcup B} = \overline{A} U \overline{B}$

   If a closed set A is defined to be one for which $A = \overline{A}$, then show that the class of all complements of closed sets is a topology on X whose closure operation is precisely that initially given.

2. (a) Show that a metric space is sequentially compact if and only if it has the Bolzano Weierstress property.

   (b) Show that every sequentially compact metric space is sequentially metric space is totally bounded.
3. State and prove Ascoli’s theorem.
4. State and prove Urysohn’s Lemma.
5. a Show that continuous image of a connected .

   b) Prove that $\mathbb{R}^n$ and $\mathfrak{C}^n$are connected space is connected.

PART – B

(Essay Answer Type)

6. Let M be a closed linear subspace of a normed linear space N. If the norm of a coset x+M in the quotient space $\frac{N}{M}$ is defined by space is $\|x+M\| = imf\{\|x+m\| : m \in M\}$ then show that $\frac{N}{M}$ is a normed linear space. Further, Banach space show that $\frac{N}{M}$ is a Banach space.
7. State and prove Hahn-Banach Theorem.
8. a) State and prove Schwarz inequality.

   b) Show that inner product in a Hilbert space is jointly continuous.
9. If M is a closed linear subspace of a Hilbert. Space H then show that $H = M \oplus M^{\perp}$.
10. If $\{e_i\}$ is an orthonormal set in a Hilbert space H then show that

    $x - \sum (x, e_i) e_j \perp e_i$ for each j.

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Code No. 7961/CDE/N

FACULTY OF SCIENCE

M.Sc. (Final) CDE Examinations, July 2019

Subject: Mathematics
Paper: II - Measure Theory

Time: 3 Hours **Max. Marks: 80**

PART – A (5 x 4 = 20 Marks)
(Short Answer Type)

Note: Answer any five of the following questions. Each question carries 4 marks.

1. Suppose $\{E_j\}$ is a sequence of measurable sets such that $E_{j+1} \subset E_j \forall j$ and $m(E1) < \infty$ then prove that $m\left(\bigcap_{n=1}^{\infty} E\right) = \lim_{n\to\infty} m(E_n)$
2. Suppose f, g are bounded measurable functions defined on a measurable set E of finite measure prove that $\int_E (af + bg) = a\int_E f + b\int_E g$ for all constants a, b
3. Define positive variation P, Negative variation N and total variation T of a real valued function f defined on [a, b] prove that P+N =T
4. Suppose f is absolutely continuous on [a, b] prove that f is a function of bounded variation on [a, b]
5. State and prove Monotone convergence theorem.
6. Define a signed measure on a measurable space (x, β). Suppose(x, β, μ) is a measure space and f is an integrable function on X with respect to μ. Prove that the function $\nu : \beta \to \mathbf{R}$ defined by $\nu(E) = \int_E f \, d«\ \forall\ E \in$ is a signed measure on β.
7. Suppose μ^* is an outer measure on P(X) and β is the class of all μ^*-measurable sets. Prove that $\bar{«}$ the restriction of μ^* to β is a complete measure onβ .
8. If $E \subset F$ prove that $«_*(E) \le «_*(F)$.

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PART – B (5x12 = 60 Marks)
[Essay Answer Type]

9. a) State and prove Egoroff's theorem.

OR

b) Suppose f is a non-negative measurable function which is integrable over a measurable set E. Prove that given any $\in$>0 there exists a $¢ > 0$ such that for all subsets $A \subset E$ with m(A)< δ we have $\int_A f < \in$

10. a) suppose f is a bounded measurable function defined on [a, b]. Suppose F(x) is defined by $F(x) = F(a) + \int_a^x f(t)dt$ for all $x \in [a,b]$. Prove that $F' = f.a.e$

OR

b) i) Prove that L^p [0, 1] is a linear space.

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ii) State and Prove Minkowski's inequality.

11. a) State and prove Fatou' s Lemma in a measure space (X, , «).

OR

b. State and prove Lebesgue decomposition theorem.

12. a) State and prove Fubini's theorem.

OR

b) Suppose $E \subset X$ with $\mu^*(E) < \infty$. Prove that E is μ^*-measurable if and only if
$« *(E) = «_*(E)$

13. a) Define a lobesgue measurable function.

Suppose f is an extended red valued function defined on a measurable subset E of **R**, prove that the following are equivalent.

i) $\{x \in E : f(x) > \sim\}$ is measurable for any $\alpha \in$ **R**

ii) $\{x \in E : f(x) \geq \sim\}$ is measurable for any $\alpha \in$ **R**

iii) $\{x \in E : f(x) < \sim\}$ is measurable for any $\alpha \in$ **R**

iv) $\{x \in E : f(x) \leq \sim\}$ is measurable for any $\alpha \in$ **R.**

b) Prove that the class β of all μ^*- measurable sets is a σ algebra of subsets of X.

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FACULTY OF SCIENCE

M.Sc. (Final) CDE Examinations, July 2019

Subject: Mathematics

Paper – II : Measure & Integration

Time: 3 Hours **Max. Marks: 100**

PART – A

(Short Answer Type)

Note: Answer any five of the following questions, choosing atleast two from each Part. All questions carry equal marks.

1. a) Suppose $\mathcal{A}$ is a collection of subsets of X satisfying

   i) A$\in\mathcal{A}\Rightarrow \overline{A}\in A$

   ii) A, B $\in\mathcal{A}\Rightarrow$ A $\cap$ B $\in\mathcal{A}$ $\Rightarrow$ A - B$\in\mathcal{A}$

   Prove that A is an algebra of sets in X. Also prove that A, B $\in\mathcal{A}$

   b) Suppose $\mathcal{A}$ is an algebra of sets in X. Suppose $\{A_n\}$ is a sequence of sets in $\mathcal{A}$.

   Prove that there exists a sequence $\{B_n\}$ of sets in $\mathcal{A}$ such that

   i) $\overset{\infty}{\underset{n=1}{U}} B_n == \overset{\infty}{\underset{n=1}{U}} A_n$

   ii) $B_n \bigcap B_m = \ddot{}\ \ for \quad n \neq m$

2. a) Prove that every Borel set is measurable.

   b) Suppose $\{f_n\}$ is a sequence of measurable functions, defined on a measurable set E in $\mathbb{R}$. Prove that

   i) $\sup\limits_n t_n$ ii) $\inf\ f_n$ iii) $\lim\sup f_n$ iv) lim inf f are all measurable functions.

3. a) Define Lebesque outer measure m*. Prove that m*. is countably sub additive.

   b) Suppose E $\subseteq$ **R**. Prove that the following are equivalent

   i) E is measurable

   ii) Given any $\in$> 0 there exists a closed set F such that F$\subset$E and m*(E-F)=0

4. a) Suppose f is a bounded function defined on a measurable set E of finite measure. Prove that f is measurable if and only if

$$\sup_{\substack{¤\leq f \\ ¤ simple}} \int_E ¤\ dx = \inf_{\substack{\cdot\ \geq f \\ \cdot\ simple}} \int_E \cdot\ \ dx$$

5. a) Suppose ϕ, ψ are simple functions which vanishes outside a set of finite measure. Prove that.

$$\int (a¤ + b\cdot\ ) = a\int ¤ + b\int \cdot\ \ dx\ .$$

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b. Suppose $\{u_n\}$ is a sequence of non-negative measurable functions defined on a measurable set E such that $\int_E f = \sum_{n=1}^{\infty} \int_E u_n$

PART – B
(Essay Answer Type)

6. a) Suppose f is integrable on [a, b] and $F(x) = \int_a^x f(t)\,dt \ \forall x \in [a,b]$ prove that F is a continuous function of bounded variation on [a, b]

b) Suppose f is integrable on [a, b] and $F(x) = F(a) + \int_a^x f(t)\,dt \ \forall x \in [a,b]$ prove that $F' = f \ a.e.$

7. Prove that a normed linear space X is complete if and only if every absolutely summable series is summable.

8. a) Suppose(X, β, «) is a measure space and f, g are non-negative measurable functions defined on E∈ β. Prove that
$\int_E (af + bg) = a\int_E f + b\int_E g \ \ \forall$ non-negative constants a, b.

b) State and prove Lebesque-Convergence theorem.

9. State and prove the Radon-Nikodym theorem.

10. Define an outer measure μ* and a μ*-measurable set ; Prove that the class β of all μ*-measurable sets is a σ-algebra of sets.

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Code No.7976/CDE

FACULTY OF SCIENCE

M.Sc. (Final) CDE Examination, June/July 2019

Subject : Statistics

Paper – II : Linear Models and Design of Experiments

Time : 3 Hours **Max. Marks: 80**

Note: Answer any five questions from Part – A and all questions from Part – B.

PART – A (5x4 = 20 Marks)
(Short Answer Type)

1. State and prove the necessary and sufficient conditions for estimability of a linear parametric function.
2. Explain the need for generalization of Linear Models.
3. Explain the basic principles of Design of Experiments.
4. Explain the analysis of CRD.
5. Explain the analysis of 2^2 factorial experiments.
6. Distinguish between Total confounding and Partial confounding.
7. What is BIBD? Explain.
8. What is Youden square design? Explain.

PART – B (5x12=60 Marks)
(Essay Answer Type)

9. (a) Show that the number of linear unbiased estimates (l.u.e's) of a linear parametric function (l.p.f) is either only one or infinitely many.

OR

(b) State and prove Gauss-Markov Theorem.

10. (a) Explain the analysis of covariance for one way classification.

OR

(b) Explain the analysis of LSD.

11. (a) Explain the analysis of 3^2 factorial experiments.

OR

(b) What are split-plot designs? When do you recommend the use of such designs? Outline the method of analysis of a split-plot design for two factors A and B having α and β levels respectively. Give the expressions for standard errors of differences of treatment means.

12. (a) Define simple lattice design with an example. Outline the analysis of this design.

OR

(b) When an incomplete block design is said to be balanced? For a balanced incomplete block design with the usual notations, prove that bk=vr, r(k-1)=λ(v-1) and b$\geq$v.

13. (a) Explain the analysis of RBD with one missing observation.

OR

(b) Construct one-quarter replicate of 2^5 factorial experiment with defining relations I_1= +ABCD and I_2= +ACE. Give the alias structure and analysis of this design.

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Code No. 7963/CDE/N

FACULTY OF SCIENCE

M.Sc. (Final) CDE Examination, July/August 2019

Subject : Mathematics
Paper – IV : Fluid Mechanics

Time : 3 Hours **Max. Marks: 80**

Note: Answer all questions from Part – A and Part – B. Each question Carries 4 marks in Part – A and 12 marks in Part – B.

PART – A (5x4 = 20 Marks)
(Short Answer Type)
Note: Answer any five of the following questions. Each question carries four marks.

1. Describe Eulerian method.
2. Define centre of mass. Find centre of mass of solid hemisphere of radius a.
3. Discuss general motion of a cylinder in two dimensions.
4. Define elliptic co-ordinates.
5. Write elementary properties of Vortex Motion.
6. Show that every Vortex is always composed of the same elements of the fluid.
7. Define Reynold's number and write its significances.
8. Write a short note on Prandtl's boundary layer theory.

PART – B (5x12=60 Marks)
(Essay Answer Type)
Note: Answer all the following questions by using internal choice. Each question carries 12 marks.

9. (a) State and prove Parallel axis theorem, using it find the moment of inertia of a uniform circular disc about an axis in the plane of the disc and tangent to the edge.

OR

(b) Derive equation of continuity in cylindrical coordinates.

10. (a) Discuss steady flow between two co-axial cylinders.

OR

(b) State and prove Blasius theorem.

11. (a) Discuss steady flow of an incompressible viscous fluid through a tube of elliptic cross section under constant pressure gradient.

OR

(b) Discuss Motion of sphere through a liquid at rest.

12. (a) Discuss unsteady flow of Viscous incompressible fluid over a suddenly accelerated flat plate.

OR

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(b) Write short notes on the following
   (i) Dynamical similarity.
   (ii) Boundary layer thickness.
   (iii) Energy thickness.

13. (a) Derive Navier-Stoke's equations.

OR

(b) Show that $\frac{x^2}{a^2}\tan^2 t + \frac{y^2}{b^2}\text{cat}^2 t=1$ is a possible form of the boundary surface of a liquid and find an expression for the normal velocity.

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Code No. 7968/CDE/BL

FACULTY OF SCIENCE

M.Sc. (Final) CDE Examination, July/August 2019

Subject : Mathematics

Paper – IV : Integral Transforms Integral Equations & Calculas of Variations

Time : 3 Hours **Max. Marks: 100**

Note: Answer any Five questions choosing atleast two from each part.
All questions carry equal marks.

PART – A

1. (a) Fiond the Laplace transforms of G(t), where $G(t) = \begin{cases} e^{t-a}, & t > a \\ 0, & t < a \end{cases}$.

   (b) Evaluate $L^{-1}\left\{\dfrac{3p-2}{p^2+4p+20}\right\}$.

   (c) Solve $[tD^2+(1-2t)D-2]y=0$ if $y(0)=1$, $y'(0)=2$.

2. (a) Find Fourier sine transform of $\dfrac{e^{-ax}}{x}$.

   (b) Find the Hankel transform of e^{-ax} taking x $J_0(px)$ as the Kernel of the transformation.

3. (a) Form integral equation corresponding to the differential equation
   $y''+(1+x^2)y = \text{Cos } x$, $y(0) = y'(0)=0$.

4. (a) Solve the integral equation $\text{¨}(x) = e^2 + 2\int_0^x \cos(x-t)\,\text{¨}(t)dt.$

   (b) Solve the integral equation $\int_0^x \cos(x-t)\,\text{¨}(t)dt = x + x^2$.

5. (a) Solve the integral equation $\text{¨}(x) = \int_0^1 \cos(xt)\,\text{¨}^3(t)dt.$

   (b) Solve $\text{¨}(x) = Cos\ x + \text{a}\int_0^{\text{®}} \sin(x-t)\,\text{¨}(t)dt$ with degenerate Kernel.

PART – B

6. (a) Solve the homogeneous integral equation
   $\text{¨}(x) = \text{a}\int_0^{\text{®}} Cos(x+t)\,\text{¨}(t)dt = 0$.

   (b) Define Green's function and give its four properties.

7. (a) Derive the necessary condition for the functional $\neg[y(x)] = \int_a^b F(x,y,y')\ dx$ with boundary conditions $y(a)=y_a$, $y(b)=y_b$ to have an extremum.

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(b) Solve the variational problem $\int_0^e (xy' + yy')dx,\ y(0) = 0,\quad y(e) = 1.$

8. (a) Find the external of the functional V[x(t), y(t)]= $\int_0^1 (y'^2 + x'^2 + 2y)dx$, subject to the conditions x(0)=0, x(1)=1, y(0)=1, y(1)=3/2.
   (b) State the isoperimetric problem and obtain its solution using principle of variational calculus.

9. (a) Find the extremats of the functional V[y(x), z(x)]= $\int_0^{\text{®}/2} [y'^2 + z'^2 + 2yz)dx$, y(0)=0=z(0), $y\left(\text{®}\!\!\diagup_2\right) = 1 = -z(\text{®}\!\!\diagup_2)$.
   (b) Prove that the sphere is the solid figure of revolution, which for given surface area, has maximum volume.

10. (a) State and prove Hamilton's principle, using it derive Lagrange equation of Motion.
    (b) Derive the equations of Motion of a projectile in space using Hamilton's equations.

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OU - coe OU - coe

Code No.7978/CDE

FACULTY OF SCIENCE

M.Sc. (Final) CDE Examination, July/August 2019

Subject : Statistics

Paper – IV : Time Series Analysis Statistical Process & Quality Control

Time : 3 Hours **Max. Marks: 80**

Note: Answer any Five of the following in not exceeding 20 lines each.

PART – A (5x4 = 20 Marks)
(Short Answer Type)

1. Define Time series and give some real life examples.
2. How does a time series model identified using Auto correlation function and Partial auto correlation function.
3. Explain briefly the diagnostic checking of Time series model.
4. How do you obtain initial estimates of parameters for AR(p) model.
5. Explain the need of implementing control charts during production.
6. Explain the construction of R-chart.
7. What is an ideal OC-curve. How do you mark the producer's risk and consumer's risk on an ideal occurrence?
8. Define double sampling plan, write the advantages of double sampling plan over single sampling plan.

PART – B (5x12=60 Marks)
(Essay Answer Type)

9. (a) Explain the duality between AR and MA process.

OR

(b) Define power spectrum. Explain the estimation of power spectrum.

10. (a) Derive the minimum Mean square error forecasts.

OR

(b) Derive the MLE's of AR(p) model.

11. (a) What is the difference between C-chart and u-chart. Obtain the O.C. function of C-chart.

OR

(b) What is CUSUM chart. How do you calculate ARL in CUSUM chart.

12. (a) Explain Dodge continuous sampling plans and its properties.

OR

(b) Define ATI, Derive the formula for ATI.

13. (a) Derive the control limits for np-chart, when 'n' is varying.

(b) Derive the power spectrum and variance for ARMA (1, 1) model.

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OU - cde OU - cde

Code No. 7964/CDE/N

FACULTY OF SCIENCE

M.Sc. (Final) CDE Examinations, July 2019

Subject: Mathematics

Paper - V : Integral Transforms Integral Equations & Calculus of Variations

Time: 3 Hours **Max. Marks: 80**

PART – A (5 x 4 = 20 Marks)
(Short Answer Type)

Note: Answer any five the following questions in not exceeding 20 lines each.

1. Evaluate L $\left\{e^{-2t}(3\cos 6t - 5\sin 6t)\right\}$
2. Find the finite Fourier Sine and cosine transform of $f(x) = x \quad in\ 0 < x < ®$.
3. Verify that $¨(x) = \frac{1}{®\sqrt{x}}$ is a solution of $\int_0^x \frac{¨(t)}{\sqrt{x-t}}dt = 1$
4. Solve the integral equation $¨(x)\sin x + 2\int_o^x e^{x-t}¨(t)dt$ by means of resolvent Kernels.
5. Find the resolvent Kernel for $k(x,t) = e^{x+t} \quad a = 0, \ b = 1$
6. Solve the following integral equation with degenerate Kernel.

$$¨(x) - 4\int_0^{\frac{®}{2}} \sin^2 x ¨(t)dt = 2x - ®$$

7. Show that the arc length of the curve.

$l[y(x)] = \int_{x_0}^{x_1} \sqrt{1 + y^{12}}dx$ has externals on the straight line $y = C_1 x + C_2$

8. Find an extremum for the functional $\gamma[y(x)] = \int_{x_o}^{x_1}[y'' + 12xy]dx$ subject to $\int_{x_o}^{x_1} 6xdx$

PART – B (5 x 12 = 60 Marks)
(Essay Answer Type)

9. a) State and prove Convolution theorem for inverse Laplace transforms.

OR

b) State and prove convolution theorem for Fourier transforms.

10. a) Define Beta function and show that

(i) $(m,n) = \ (n,m) \ for \ m,n > 0$

(ii) $(m,n+1) + \ (m+1,n) = \ (\mathrm{m},\mathrm{n})$

OR

b) Illustrate the method for constructing a green’s function for Second-order differential equation of the form.

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$$[p(x)y']' + q(x)y = 0 \quad \text{with boundary conditions } y(a) = y(b) = 0$$
$$p(x) \neq 0 \, on[a,b] \;\&\; p(x) \in C^{(1)}[a,b]$$

11. a) Solve the integral equation $¤(n) - ª \int_{-®}^{®} (x\cos t + t^2 \sin x + \cos x \sin t)¤(t)dt = x$

OR

b) Solve the following Integro-Differential Equation $¤''(x) + \int_{o}^{x} e^{2(x-t)}¤'(t)dt = e^{2x}, ¤(0) = 0$, $¤'(0) = 1$

12. a) Solve $¤(x) - ª \int_{-1}^{1} (5xt^3 + 4x^2t + 3xt)¤(t)dt = 0$

OR

b) Show that $(m,n) = 2\int_{0}^{\frac{®}{2}} Sin^{2m-1}x Cos^{2n-1}x dx$ hence evaluate $\int_{0}^{\frac{®}{2}} \sin^4 x \cos^6 x \, dx$.

13. a) Explain Brachitochrome problem and find a variational solution to it.

OR

(b) Derive Euler-Ostrogradsky Equation for the function $v[z(x,y)] = \iint_D F\left(x, y, z, \frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}\right) dxdy.$

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Code No. 7979/CDE

FACULTY OF SCIENCE

M.Sc. (Final Practical) CDE Examination, July 2019

Subject : Statistics

Paper – I : Statistical Inference, Linear Models & Design of Experiments

Time : 3 Hours **Max. Marks: 100**

Note: Answer any three questions. All questions carry equal marks.
(Scientific calculators are allowed)

1. (a) Five measurements of the tor content of a certain kind of cigarette yielded 14.5, 14.2, 14.4, 14.3 an d14.6 mg per cigarette. Show that the difference between the mean of this sample and the average tar claimed by the manufacturer, μ=14.0 is significant at α=0.05. Assume normality.
   (b) From the following data examine by using a sequential test procedure whether the coin is unbiased against the alternative that the probability of a head is 0.6. Take α=0.01 and β=0.1. For 30 trials the outcomes are : H T T H H H T H T H H H T H H T T H H H H T H T T H H T T T.

2. (a) The following random samples are measurements of the heat-producing capacity (in millions of calories per ton) of specimens of coal from two mines:
   Mine 1: 8260 8130 8350 8070 8340
   Mine 2: 7950 7890 7900 8140 7920 7840
   Assuming the two populations from which the samples are drawn to be $N(«_1,±^2)$ and $N(«_2,±^2)$ respectively, test $H_0 : «_1 = «_2$ Vs $H_1 : «_1 \neq «_2$ at $~ = 0.01$ level.
   (b) The following sequence of yes 'Y' and No. 'N' was obtained by individuals according to whether they are graduates or not : N N N N Y Y N Y Y N Y N N N N . Test for randomness of the series at 0.1 level.

3. Using the following data, test whether there is any association between disability of workers with their work performance. Test at $~ = 0.05$.

		Performance		
		Above average	Average	Below average
	Blind	21	64	17
Disability	Deaf	16	19	14
	No disability	29	93	28

4. The following are sample data provided by a moving company on the weights of six shipments, the distances they are moved, and the damage that was incurred:

Weight(in 1000 pounds) X_1:	4.0	3.0	1.6	1.2	3.4	4.8
Distance(1000 miles) X_2:	1.5	2.2	1.0	2.0	0.8	1.6
Damage(Dollars) Y:	160	112	69	90	123	186

   a) Fit least squares linear regression of Y on X_1 and X_2.
   b) Test the hypothesis of overall goodness of fit.
   c) Test the hypothesis of the coefficient of X_2 is zero.

5. Seven different hardwood concentrations (A, B, C, D, E, F, G) are being studied to

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determine the affect on the strength of the paper produced. However the plant can produce only three runs. The data was observed for a Youden square design.

Days(Blocks)

		1	2	3	4	5	6	7
Columns	1	A 114	B 120	C 117	D 149	E 210	F 119	G 117
	2	B 126	C 137	D 129	E 150	F 143	G 123	A 134
	3	D 141	E 145	F 120	G 136	A 118	B 118	C 127

(i) Carryout analysis to test the hypothesis of equality of mean strength for the seven hardwood concentrations.
(ii) Estimate the standard error of the difference between any two treatment effects.

6. An experiment was carried out to study the effect of N: nitrate (n_0, n_1), P: Potash (p_0, p_1) and phosphate K(k_0, k_1) each at two levels on the yield (in tons) of potatoes using two replicates with incomplete blocks confounding, one replicate is one replicate each:

Replicate-I

Block 1	Block 2
Npr 68	(1) 67
n 70	np 75
p 90	nk 59
k 80	Pk 56

Replicate-II

Block 1	Block 2
pr 78	np 64
npr 65	nr 77
(1) 55	p 102
n 48	k 58

Identify the confounded effects and carryout the analysis accordingly.

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